

Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 10

Time Allowed: 3 hours

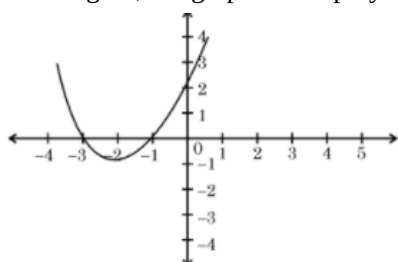
Maximum Marks: 80

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. The total number of factors of a prime number is: [1]
- a) 2 b) 1
- c) 3 d) 0
2. In the figure, the graph of the polynomial $p(x)$ is given. The number of zeroes of the polynomial is: [1]



3. A system of linear equations is said to be inconsistent if it has



-

- www.studentbro.in

-  www.studentbro.in

a) $\frac{7}{9}$

b) $\frac{5}{9}$

c) $\frac{3}{9}$

d) $\frac{1}{9}$

18. If the difference of mode and median of a data is 24, then the difference of median and mean of the same data is: [1]

a) 8

b) 12

c) 34

d) 24

19. **Assertion (A):** A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3 . [1]

Reason (R): To calculate the volume of vessel the expression used here is $v = \pi r^2 h + \frac{4}{3} \pi r^3$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The 11th term of an AP is 7, 9, 11, 13 is 67. [1]

Reason (R): If s_n is the sum of first n terms of an AP then its nth term a_n is given by $a_n = s_n - s_{n-1}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

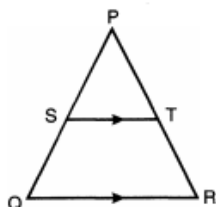
c) A is true but R is false.

d) A is false but R is true.

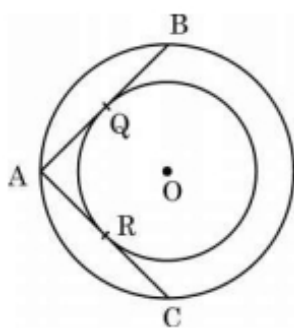
Section B

21. Find H.C.F. and L.C.M. of 56 and 112 by prime factorisation method. [2]

22. In the given figure, in a triangle PQR, $ST \parallel QR$ and $\frac{PS}{SQ} = \frac{3}{5}$, $PR = 28 \text{ cm}$, find PT. [2]



23. In Fig., there are two concentric circles with centre O. If ARC and AQB are tangents to the smaller circle from the point A lying on the larger circle, find the length of AC, if $AQ = 5 \text{ cm}$. [2]



24. Find the value of : $\sin 30^\circ \cdot \cos 60^\circ + \cos 30^\circ \cdot \sin 60^\circ$. [2]

Is it equal to $\sin 90^\circ$ or $\cos 90^\circ$?

OR

If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, prove that $a^2 + b^2 = m^2 + n^2$

25. Find the area of a quadrant of a circle whose circumference is 22 cm. [2]

OR

What is the diameter of a circle whose area is equal to the sum of the areas of two circles of diameters 10 cm and 24

cm.

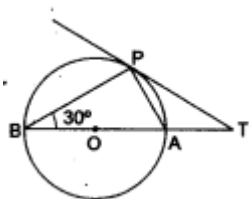
Section C

26. Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively. [3]
27. Find the quadratic polynomial, sum and product of whose zeroes are -1 and -20 respectively. Also find the zeroes of the polynomial so obtained. [3]
28. If the last term of an A.P. of 30 terms is 119 and the 8th term from the end (towards the first term) is 91, then find the common difference of the A.P. Hence, find the sum of all the terms of the A.P. [3]

OR

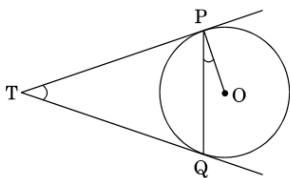
The sum of 5th and 9th terms of an A.P. is 72 and the sum of 7th and 12th terms is 97. Find the A.P.

29. In the given figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If $\angle PBT = 30^\circ$, prove that BA: AT = 2:1. [3]



OR

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



30. Prove that $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$ [3]
31. If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2 . [3]

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

Section D

32. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If, the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article. [5]

OR

If (-5) is a root of the quadratic equation $2x^2 + px + 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k.

33. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30° , which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60° . Find the further time taken by the car to reach the foot of the tower from this point. [5]
34. A solid is in the shape of a right-circular cone surmounted on a hemisphere, the radius of each of them being 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid. [5]

OR

A tent is in the shape of a right circular cylinder up to a height of 3 m and then a right circular cone, with a maximum

height of 13.5 m above the ground. Calculate the cost of painting the inner side of the tent at the rate of ₹ 2 per square metre, if the radius of the base is 14 m.

35. A survey regarding the heights (in cm) of 50 girls of X of a school was conducted and the following data was obtained: [5]

Height (in cm)	120 - 130	130 - 140	140 - 150	150 - 160	160 - 170	Total
Number of girls	2	8	12	20	8	50

Find the mean and mode of the above data.

Section E

36. Read the following text carefully and answer the questions that follow: [4]

Two schools **P** and **Q** decided to award prizes to their students for two games of Hockey ₹ x per student and Cricket ₹ y per student. School **P** decided to award a total of ₹ 9,500 for the two games to 5 and 4 students respectively; while school **Q** decided to award ₹ 7,370 for the two games to 4 and 3 students respectively.



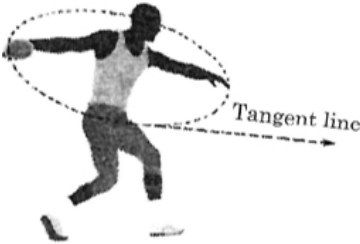
- i. Represent the following information algebraically (in terms of x and y). (1)
- ii. What is the prize amount for hockey? (1)
- iii. Prize amount on which game is more and by how much? (2)

OR

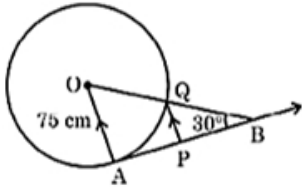
What will be the total prize amount if there are 2 students each from two games? (2)

37. Read the following text carefully and answer the questions that follow: [4]

The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.



In the given figure, AB is one such tangent to a circle of radius 75 cm. Point O is centre of the circle and $\angle ABO = 30^\circ$. PQ is parallel to OA .



- i. find the length of AB . (1)
- ii. find the length of OB . (1)
- iii. find the length of AP . (2)

OR

find the length of PQ. (2)

38. **Read the following text carefully and answer the questions that follow:**

[4]

Using Cartesian Coordinates we mark a point on a graph by how far along and how far up it is.

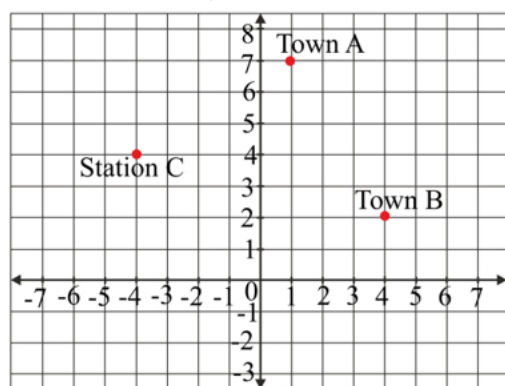
The left-right (horizontal) direction is commonly called X-axis.

The up-down (vertical) direction is commonly called Y-axis.

When we include negative values, the x and y axes divide the space up into 4 pieces.

Read the information given above and below:

Two friends Veena and Arun work in the same office in Delhi. In the Christmas vacations, both decided to go their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi.



- Who will travel more distance to reach their home? (1)
- Find the location of the station. (1)
- Find in which ratio Y-axis divide Town B and Station. (2)

OR

Find the distance between Town A and Town B. (2)

Solution

Section A

1. (a) 2

Explanation: The total number of factors of a prime number = 2 i.e. 1 and itself

2. (a) 2

Explanation: 2

The number of zeroes is 2 as the graph does cut the x-axis 2 times.

3.

(d) no solution

Explanation: A system of linear equations is said to be inconsistent if it has no solution means two lines are running parallel and not cutting each other at any point.

4.

(c) 4

Explanation: For a quadratic equation to be a perfect square

$$D = 0$$

$$b^2 - 4ac = 0$$

$$(4)^2 - 4(1)(\lambda) = 0$$

$$16 - 4\lambda = 0$$

$$16 = 4\lambda$$

$$\lambda = 4$$

5. (a) 22

Explanation: Given, $a_1 = 14$, $t_n = 119$

$$d = a_2 - a_1 = 19 - 14 = 5$$

$$t_n = a_1 + (n - 1)d$$

$$119 = 14 + (n - 1)5$$

$$119 - 14 = 5n - 5$$

$$105 + 5 = 5n$$

$$110 = 5n$$

$$n = 22$$

6.

(d) (2, 0)

Explanation: Let the other point on x axis is (x, 0)

By distance formula

$$\sqrt{(x + 4)^2 + (0 - 0)^2} = (6)$$

$$x + 4 = 6$$

$$x = 2$$

Hence the point is (2, 0)

7.

(c) (2, 5)

Explanation: Let coordinate of A(x,y)

Then coordinate of mid point are $\left[\frac{(x-2)}{2}, \frac{(y+3)}{2} \right]$

On comparing the coordinates of mid points

$$\frac{(x-2)}{2} = 0$$

$$x = 2$$

$$\frac{(y+3)}{2} = 4$$

$$y = 5$$

Coordinates of A are (2, 5).

8. (a) 1 cm

Explanation: In $\triangle ABO$ and $\triangle QPO$

$\angle BAO = \angle PQO$ (by alt. angle)

$\angle AOB = \angle QOP$ (vert. oppo. angle)

$\therefore \triangle ABO \sim \triangle QPO$ (by AA Similarity)

$$\therefore \frac{AB}{QP} = \frac{OB}{OP}$$

$$\frac{6}{2} = \frac{3}{OP}$$

$$OP = 1 \text{ cm}$$

9.

(c) $62\frac{1}{2}^\circ$

Explanation: Given, $\angle APB = 55^\circ$

$\therefore \angle ACB = 180^\circ - 55^\circ = 125^\circ$...($\because \angle APB$ and $\angle ACB$ are supplementary angles)

Now, as we know that

Angle subtended by an arc at the centre = $2 \times$ angle subtended by arc at any point on the remaining part of the circle

$\therefore 125^\circ = 2 \times \angle AQB$

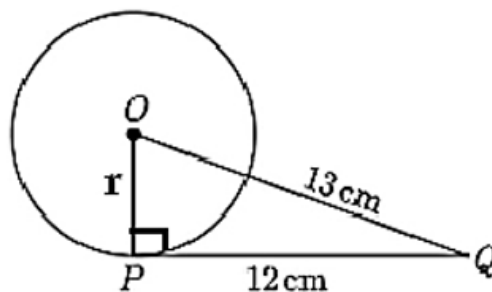
$$\Rightarrow \angle AQB = \frac{125}{2}$$

$$= 62.5^\circ \text{ or } 62\frac{1}{2}^\circ$$

10.

(b) 5

Explanation:



\therefore We know that

$PQ \perp OP$

$\therefore \triangle QPO$ is right angled \triangle

\therefore By pythog theory.

$$QO^2 = QP^2 + OP^2$$

$$(13)^2 = (12)^2 + OP^2$$

$$r^2 = 169 - 144.$$

$$r^2 = 25$$

$$r = 5 \text{ cm}$$

11. (a) $\frac{3}{2}(\sqrt{2} - 1)$

$$\text{Explanation: } = \frac{\sin 90^\circ + \cos 60^\circ}{\sec 45^\circ + \tan 45^\circ}$$

$$= \frac{1 + \frac{1}{2}}{\sqrt{2} + 1}$$

$$= \frac{\frac{3}{2}}{\sqrt{2} + 1}$$

upon rationalization

$$= \frac{3}{2} \times \frac{1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1}$$

$$= \frac{3}{2} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1}$$

$$= \frac{3}{2}(\sqrt{2} - 1)$$

12. (a) $-\frac{1}{4}$

Explanation: $(3 \sin^2 30^\circ - 4 \cos^2 60^\circ)$

$$\Rightarrow 3 \times \left(\frac{1}{2}\right)^2 - 4 \times \left(\frac{1}{2}\right)^2$$

$$\Rightarrow -\frac{1}{4}$$

13.

(d) $\frac{15}{2}$ m

Explanation:

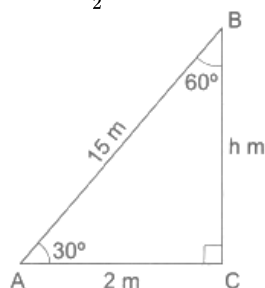
Let AB be the ladder and BC be the wall

$$\text{Then, } \angle ABC = 60^\circ \Rightarrow \angle CAB = (90^\circ - 60^\circ) = 30^\circ$$

Let BC = h m. then,

$$\frac{BC}{AB} = \sin 30^\circ \Rightarrow \frac{h}{15} = \frac{1}{2}$$

$$\Rightarrow h = \frac{15}{2}$$



14.

(b) $\frac{132}{7} \text{ cm}^2$

Explanation: Angle of the sector is 60°

$$\text{Area of sector} = \left(\frac{\theta}{360^\circ}\right) \times \pi r^2$$

$$\therefore \text{Area of the sector with angle } 60^\circ = \left(\frac{60^\circ}{360^\circ}\right) \times \pi r^2 \text{ cm}^2$$

$$= \left(\frac{36}{6}\right) \pi \text{ cm}^2$$

$$= 6 \times \left(\frac{22}{7}\right) \text{ cm}^2$$

$$= \frac{132}{7} \text{ cm}^2$$

15. (a) 13 cm

Explanation: Radius of wheel = $\frac{91}{2}$ cm

$$\text{Angle between two adjoining spokes, } \theta = \frac{360^\circ}{22}$$

$$\therefore \text{Length of arc} = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{360^\circ}{360^\circ \times 22} \times 2 \times \frac{22}{7} \times \frac{91}{2} = 13 \text{ cm}$$

16.

(b) $\frac{1}{25}$

Explanation: $n(S) = 100$

$$E = \{1, 8, 27, 64\}$$

$$n(E) = 4$$

the probability of drawing a number on the card that is a cube is

$$P(E) = \frac{4}{100} = \frac{1}{25}$$

17.

(b) $\frac{5}{9}$

Explanation: Numbers $x = 1, 2, 3$ and $y = 1, 4, 9$

$$\text{Now } xy = \{1, 4, 9, 2, 8, 18, 3, 12, 27\} = 9$$

$$\therefore n = 9$$

$$\text{and } xy < 9 \text{ are } 1, 2, 3, 4, 8$$

$$\therefore m = 5$$

$$\therefore P(xy < 9) = \frac{5}{9}$$

18.

(b) 12

Explanation: Given,

$$\text{mode} - \text{median} = 24$$

$$\text{median} - \text{mean} = ?$$

we know that,

$$\text{mode} = 3 \text{ median} - 2 \text{ mean}$$

$$\text{mode} = \text{median} + 2 \text{ median} - 2 \text{ mean}$$

$$\text{mode} - \text{median} = 2 \text{ median} - 2 \text{ mean}$$

$$24 = 2 (\text{median} - \text{mean})$$

$$\text{median} - \text{mean} = \frac{24}{2} = 12$$

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20.

(c) A is true but R is false.

Explanation: A is true but R is false.

Section B

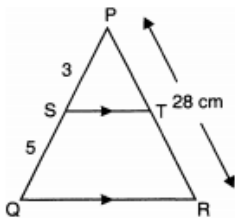
21. Using the factor tree we have,

$$56 = 2^3 \times 7 \text{ and } 112 = 2^4 \times 7$$

$$\text{Hence HCF is } 2^3 \times 7 = 56 \text{ and}$$

$$\text{LCM is } 2^4 \times 7 = 112$$

22. According to the question,



$$ST \parallel QR$$

$$\therefore \frac{PS}{PQ} = \frac{PT}{PR} \text{ (By BPT)}$$

$$\text{Given, } \frac{PS}{SQ} = \frac{3}{5} \text{ and } PR = 28 \text{ cm}$$

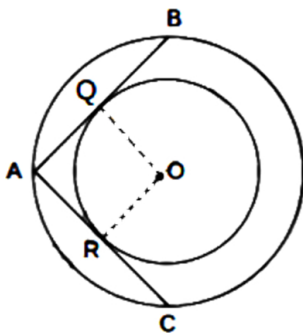
$$PQ = 3 + 5 = 8$$

$$\text{or, } \frac{PS}{PQ} = \frac{PT}{PR}$$

$$\text{or, } \frac{3}{8} = \frac{PT}{28}$$

$$\therefore PT = \frac{3 \times 28}{8} = 10.5 \text{ cm}$$

23. Here, AC and AB are the tangents from external point A to the smaller circle.



$$\therefore AC = AB$$

Now, AB is the chord of bigger circle and OQ is the perpendicular bisector of chord AB.

$$\therefore AQ = QB$$

$$\text{or, } AB = 2AQ$$

$$\text{or, } AB = 2(5) = 10 \text{ cm ...[}\therefore \text{ Given } AQ = 5 \text{ cm]}$$

$$\text{or, } AC = 10 \text{ cm}$$

$$24. \sin 30^\circ \cdot \cos 60^\circ + \cos 30^\circ \cdot \sin 60^\circ$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4}$$

$$= \frac{4}{4}$$

$$= 1 = \sin 90^\circ = \cos 90^\circ$$

OR

Given,

$$\text{R.H.S} = m^2 + n^2$$

$$= (a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2 \text{ [since, } m = a \cos \theta + b \sin \theta \text{ and } n = a \sin \theta - b \cos \theta]$$

$$= (a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \sin \theta) + (a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta) \text{ [}\therefore (a \pm b)^2 = a^2 + b^2 \pm 2ab]$$

$$= a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = a^2 + b^2 = \text{L.H.S} \text{ [}\therefore \sin^2 \theta + \cos^2 \theta = 1]$$

$$\text{therefore, } m^2 + n^2 = a^2 + b^2$$

Hence proved.

25. Let the radius of the circle be r cm.

Then, circumference of the circle = $2\pi r$ cm

According to the question,

$$2\pi r = 22$$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 22$$

$$\Rightarrow r = \frac{22 \times 7}{2 \times 22} \Rightarrow r = \frac{7}{2} \text{ cm}$$

For a quadrant of a circle,

$$\text{Area} = \frac{1}{4} \pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{8} \text{ cm}^2$$

OR

Let the radius of the large circle be R.

Then, we have

Area of large circle of radius R = Area of a circle of radius 5 cm + Area of a circle of radius 12 cm

$$\Rightarrow \pi R^2 = (\pi \times 5^2 + \pi \times 12^2)$$

$$\Rightarrow \pi R^2 = (25\pi + 144\pi)$$

$$\Rightarrow \pi R^2 = 169\pi$$

$$\Rightarrow R^2 = 169$$

$$\Rightarrow R = 13 \text{ cm}$$

$$\Rightarrow \text{Diameter} = 2R$$

$$= 26 \text{ cm}$$

Section C

26. We have to find the greatest number that divides 445, 572 and 699 and leaves remainders of 4, 5 and 6 respectively. This means when the number divides 445, 572 and 699, it leaves remainders 4, 5 and 6. It means that

$$445 - 4 = 441,$$

$$572 - 5 = 567$$

$$\text{and } 699 - 6 = 693$$

are completely divisible by the required number.

For the highest number which divides the above numbers we need to calculate HCF of 441, 567 and 693.

Therefore, the required number is the H.C.F. of 441, 567 and 693 Respectively.

First, consider 441 and 567.

By applying Euclid's division lemma, we get

$$567 = 441 \times 1 + 126$$

$$441 = 126 \times 3 + 63$$

$$126 = 63 \times 2 + 0.$$

Therefore, H.C.F. of 441 and 567 = 63

Now, consider 63 and 693

again we have to apply Euclid's division lemma, we get

$$693 = 63 \times 11 + 0.$$

Therefore, H.C.F. of 441, 567 and 693 is 63

Hence, the required number is 63. 63 is the highest number which divides 445,572 and 699 will leave 4,5 and 6 as remainder respectively.

27. Sum of the zeroes: $(2 + \beta) = (-1)$

Product of the zeroes : $2\beta = -20$

So, required Quadratic polynomial

$$= [x^2 + (\alpha + \beta)x + 2\beta]$$

$$= [x^2 + (-1)x + (-20)]$$

$$= x^2 - x - 20$$

$$\Rightarrow x^2 - x - 20 = 0 \text{ is the polynomial}$$

28. Given, last term, $l = 119$

No. of terms in A.P. = 30

8th term from the end = 91

Let d be a common difference and assume that the first term of A.P. is 119 (from the end)

Since the n_{th} term of AP is

$$a_n = l + (n - 1)d$$

$$\therefore a_8 = 119 + (8 - 1)d$$

$$\Rightarrow 91 = 119 + 7d$$

$$\Rightarrow 7d = 91 - 119$$

$$\Rightarrow 7d = -28$$

$$\Rightarrow d = -4$$

Now, this common difference is from the end of A.P.

So, the common difference from the beginning = $-d$

$$= (-4) = 4$$

Thus, a common difference for the A.P. is 4.

Now, using the formula

$$I = a + (n - 1)d$$

$$\Rightarrow 119 = a + (30 - 1)4$$

$$\Rightarrow 119 = a + 29 \times 4$$

$$\Rightarrow 119 = a + 116$$

$$\Rightarrow a = 119 - 116$$

$$\Rightarrow a = 3$$

Hence, using the formula for the sum of n terms of an A.P.

$$\text{i.e., } S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_{30} = \frac{30}{2} [2 \times 3 + (30 - 1) \times 4]$$

$$= 15(6 + 29 \times 4)$$

$$= 15 \times 122$$

$$= 1830$$

Therefore, the sum of 30 terms of an A.P. is 1830

OR

It is given that the sum of 5th and 9th terms of an A.P. is 72 and the sum of 7th and 12th terms is 97.

Let " a " be the first term and ' d ' be the common difference of the Arithmetic progression .

It is given that $a_5 + a_9 = 72$ and, $a_7 + a_{12} = 97$

$$\Rightarrow (a + 4d) + (a + 8d) = 72 \text{ and, } (a + 6d) + (a + 11d) = 97.$$

Therefore, we have

$$\Rightarrow 2a + 12d = 72 \dots(1)$$

$$\Rightarrow 2a + 17d = 97 \dots(2)$$

Subtracting (1) from (2), we get

$$2a + 17d - 2a - 12d = 97 - 72$$

$$\Rightarrow 5d = 25$$



$$\Rightarrow d = 5$$

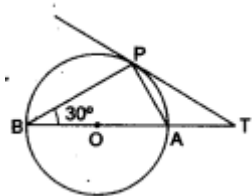
Putting $d = 5$ in (1), we get

$$2a + 60 = 72 \Rightarrow 2a = 12 \Rightarrow a = 6$$

Therefore, $a = 6$ and $d = 5$

Hence, the Arithmetic Progression $a, a+d, a+2d, \dots$ is 6, 11, 16, 21, 26,

29. O is the centre of the circle and TP is the tangent to the circle from an external point T.



From figure,

AB is the diameter

Since, angle in a semicircle is a right angle

$$\angle APB = 90^\circ$$

By using alternate segment theorem

$$\text{We have } \angle APB = \angle PAT = 30^\circ$$

Now, in $\triangle APB$

$$\angle BAP + \angle APB + \angle ABP = 180^\circ \text{ (Angle sum property of triangle)}$$

$$\angle BAP = 180^\circ - 90^\circ - 30^\circ = 60^\circ$$

Now, $\angle BAP = \angle APT + \angle PTA$ (Exterior angle property)

$$60^\circ = 30^\circ + \angle PTA$$

$$\angle PTA = 60^\circ - 30^\circ = 30^\circ$$

We know that sides opposite to equal angles are equal

$$AP = AT$$

In right triangle ABP,

$$\sin 30^\circ = \frac{AT}{BA}$$

$$\Rightarrow \frac{1}{2} = \frac{AT}{BA}$$

$$\therefore BA : AT = 2 : 1$$

OR

$TP = TQ$... (length of tangents drawn from external points)

$\therefore \angle TQP = \angle TPQ$ (angles oppo to equal sides are equal)

$OP \perp TP$ (\because at point of contact radius and tangent are \perp r)

$$\angle OPT = 90^\circ$$

$$\angle OPQ + \angle CPQ = 90^\circ$$

$$\angle TPQ = 90^\circ - \angle OPQ$$

Now, In $\triangle PTQ$

$$\angle TPQ + \angle PTQ + \angle QTP = 180^\circ$$

$$90^\circ - \angle OPQ + 90^\circ - \angle OPQ + \angle PTQ = 180^\circ$$

$$\angle PTQ = 2\angle OPQ$$

Proved.

30. First, we will show that, $\frac{1+\tan^2 A}{1+\cot^2 A} = \tan^2 A$

$$\text{LHS} = \frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}} = \frac{1+\tan^2 A}{\frac{1+\tan^2 A}{\tan^2 A}}$$

$$= (1 + \tan^2 A) \times \frac{\tan^2 A}{1+\tan^2 A}$$

$$= \tan^2 A = \text{RHS} \dots (i)$$

Now, we will show that, $\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

$$\text{LHS} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2 = \left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2$$

$$= \left[(1 - \tan A) \times \left(\frac{\tan A}{-(1-\tan A)}\right)\right]^2$$

$$= (\tan A)^2 = \tan^2 A = \text{RHS} \dots(\text{ii})$$

Hence, from (i) and (ii),

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A} \right)^2 = \tan^2 A$$

Hence proved.

31. Let f_1 and f_2 be the frequencies of class intervals 0 - 10 and 40 - 50.

$$f_1 + 5 + 9 + 12 + f_2 + 3 + 2 = 40$$

$$\Rightarrow f_1 + f_2 = 9$$

Median is 32.5 which lies in 30 - 40, so the median class is 30 - 40.

$$l = 30, h = 10, f = 12, N = 40 \text{ and } c = f_1 + 5 + 9 = (f_1 + 14)$$

$$\text{Now, median} = l + \left[h \times \frac{\left(\frac{N}{2} - c \right)}{f} \right]$$

$$\Rightarrow 32.5 = \left[30 + \left(10 \times \frac{20 - f_1 - 14}{12} \right) \right]$$

$$= \left[30 + \left(10 \times \frac{6 - f_1}{12} \right) \right]$$

$$= \left[30 + \left(\frac{30 - 5f_1}{6} \right) \right]$$

$$\frac{30 - 5f_1}{6} = 2.5$$

$$30 - 5f_1 = 15$$

$$5f_1 = 15 \Rightarrow f_1 = 3$$

$$f_1 = 3 \text{ and } f_2 = (9 - 3) = 6$$

Section D

32. Let cost of production of each article be Rs x

We are given total cost of production on that particular day = Rs 90

Therefore, total number of articles produced that day = $90/x$

According to the given conditions,

$$x = 2 \left(\frac{90}{x} \right) + 3$$

$$\Rightarrow x = \frac{180}{x} + 3$$

$$\Rightarrow x = \frac{180 + 3x}{x}$$

$$\Rightarrow x^2 = 180 + 3x$$

$$\Rightarrow x^2 - 3x - 180 = 0$$

$$\Rightarrow x^2 - 15x + 12x - 180 = 0$$

$$\Rightarrow x(x - 15) + 12(x - 15) = 0$$

$$\Rightarrow (x - 15)(x + 12) = 0 \Rightarrow x = 15, -12$$

Cost cannot be in negative, therefore, we discard $x = -12$

Therefore, $x = \text{Rs } 15$ which is the cost of production of each article.

$$\text{Number of articles produced on that particular day} = \frac{90}{15} = 6$$

OR

Since (-5) is a root of given quadratic equation $2x^2 + px + 15 = 0$, then,

$$2(-5)^2 + p(-5) - 15 = 0$$

$$50 - 5p - 15 = 0$$

$$5p = 35 \Rightarrow p = 7$$

Now $p(x^2 + x) + k = 0$ has equal roots

$$px^2 + px + k = 0$$

$$\text{So } (b)^2 - 4ac = 0$$

$$(p)^2 - 4p \times k = 0$$

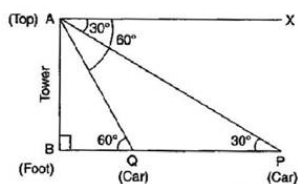
$$(7)^2 - 4 \times 7 \times k = 0$$

$$28k = 49$$

$$k = \frac{49}{28} = \frac{7}{4}$$

$$\text{hence } p = 7 \text{ and } k = \frac{7}{4}$$

33.



In right triangle ABP,

$$\tan 30^\circ = \frac{AB}{BP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BP}$$

$$BP = AB\sqrt{3} \dots\dots (i)$$

In right triangle ABQ,

$$\tan 60^\circ = \frac{AB}{BQ}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{BQ}$$

$$\Rightarrow BQ = \frac{AB}{\sqrt{3}} \dots\dots (ii)$$

$$\therefore PQ = BP - BQ$$

$$\therefore PQ = AB\sqrt{3} - \frac{AB}{\sqrt{3}} = \frac{3AB - AB}{\sqrt{3}} = \frac{2AB}{\sqrt{3}} = 2BQ \text{ [From eq. (ii)]}$$

$$\Rightarrow BQ = \frac{1}{2}PQ$$

\therefore Time taken by the car to travel a distance PQ = 6 seconds.

\therefore Time taken by the car to travel a distance BQ, i.e. $\frac{1}{2}PQ = \frac{1}{2} \times 6 = 3$ seconds.

Hence, the further time taken by the car to reach the foot of the tower is 3 seconds.

34. Given, radius of cone = radius of hemisphere

$$= r$$

$$= 7 \text{ cm}$$

$$\text{Height of cone (h)} = 2 \times \text{radius}$$

$$= 2 \times 7$$

$$= 14 \text{ cm}$$

$$\text{Volume of solid} = \text{Volume of cone} + \text{Volume of hemisphere}$$

$$\text{Volume of solid (V)} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (2r) + \frac{2}{3}\pi r^3 \dots (\because h = 2r)$$

$$= \frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3$$

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= \frac{4312}{3}$$

$$= 1437.33 \text{ cm}^3$$

OR

$$\text{Height of the cylinder} = 3 \text{ m.}$$

$$\text{Total height of the tent above the ground} = 13.5 \text{ m}$$

$$\text{height of the cone} = (13.5 - 3)\text{m} = 10.5 \text{ m}$$

$$\text{Radius of the cylinder} = \text{radius of cone} = 14 \text{ m}$$

$$\text{Curved surface area of the cylinder} = 2\pi rh \text{m}^2 = \left(2 \times \frac{22}{7} \times 14 \times 3\right) \text{m}^2 = 264\text{m}^2$$

$$\therefore l = \sqrt{r^2 + h^2} = \sqrt{14^2 + (10.5)^2} = \sqrt{196 + 110.25} = \sqrt{306.25} = 17.5$$

$$\therefore \text{Cured surface area of the cone} = \pi rl = \left(\frac{22}{7} \times 14 \times 17.5\right) \text{m}^2 = 770\text{m}^2$$

Let S be the total area which is to be painted. Then,

$$S = \text{Curved surface area of the cylinder} + \text{Curved surface area of the cone}$$

$$\Rightarrow S = (264 + 770) \text{m}^2 = 1034 \text{m}^2$$

$$\text{Hence, Cost of painting} = S \times \text{Rate} = ₹ (1034 \times 2) = ₹ 2068$$

35.

Height (in cm)	No. of girls	x_i	u_i	$f_i u_i$
120 - 130	2	125	-2	-4
130 - 140	8	135	-1	-8

140 - 150	12	145 = a	0	0
150 - 160	20	155	1	20
160 - 170	8	165	2	16
Total	50			24

Mean = $145 + \frac{24}{50} \times 10$
= 149.8

∴ mean height is 149.8 cm

Modal class is 150 - 160

Mode = $150 + \frac{(20-12)}{(2 \times 20 - 12 - 8)} \times 10$
= 154

∴ modal height is 154 cm

Section E

36. i. Given, prize amount for Hockey ₹ x and ₹ y for cricket per student

∴ Algebraic equations are

$5x + 4y = 9500$...(i)

and $4x + 3y = 7370$...(ii)

- ii. Given, prize amount for Hockey ₹ x and ₹ y for cricket per student

∴ Algebraic equations are

$5x + 4y = 9500$...(i)

and $4x + 3y = 7370$...(ii)

Multiply by 3 in equation (i) and by 4 in equation (ii)

$15x + 12y = 28,500$...(iii)

$16x + 12y = 29480$...(iv)

On subtracting equation (iii) from equation (iv), we get

$x = 980$

∴ Prize amount for hockey = ₹ 980

- iii. Given, prize amount for Hockey ₹ x and ₹ y for cricket per student

∴ Algebraic equations are

$5x + 4y = 9500$...(i)

and $4x + 3y = 7370$...(ii)

Now, put this value in equation (i), we get

$5 \times 980 + 4y = 9500$

$\Rightarrow 4y = 9500 - 4900 = 4600$

$\Rightarrow y = 1150$

∴ Prize amount for cricket = ₹ 1150

Difference = $1150 - 980 = 170$

∴ Prize amount for cricket is ₹ 170 more than hockey.

OR

Total prize amount for 2 students each from two games

$= 2x + 2y$

$= 2(x + y)$

$= 2(980 + 1150)$

$= 2 \times 2130$

$= ₹ 4260$

37. i. $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{75}{AB}$

$\Rightarrow AB = 75\sqrt{3}$ cm

- ii. $\sin 30^\circ = \frac{1}{2} = \frac{75}{OB}$

$\Rightarrow OB = 150$ cm

- iii. $QB = 150 - 75 = 75$ cm

$\Rightarrow Q$ is mid point of OB

Since $PQ \parallel AO$ therefore P is mid pint of AB

$$\text{Hence } AP = \frac{75\sqrt{3}}{2} \text{ cm.}$$

OR


$$QB = 150 - 75 = 75 \text{ cm}$$

Now, $\triangle BQP \sim \triangle BOA$


$$\Rightarrow \frac{QB}{OB} = \frac{PQ}{OA}$$

$$\Rightarrow \frac{1}{2} = \frac{PQ}{75}$$

$$\Rightarrow PQ = \frac{75}{2} \text{ cm}$$

38. i. 

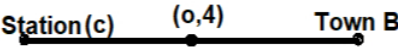
$$\begin{aligned} \text{Distance travelled by veena} &= \sqrt{1 - (-4)^2 + (7 - 4)^2} \\ &= \sqrt{5^2 + 3^2} \\ &= \sqrt{25 + 9} \\ &= \sqrt{34} \end{aligned}$$

- 

$$\begin{aligned} \text{Distance travelled by Arun} &= \sqrt{(4 - (-4))^2 + (2 - 4)^2} \\ &= \sqrt{64 + 4} \\ &= \sqrt{68} \end{aligned}$$

\therefore Arun will travel more distance to reach his home.

- ii. Location of station = (-4, 4)

- iii. 

Let y-axis divides station (c) and Town B in K : 1


$$0 = \frac{4k-4}{k+1}$$

$$4k = 4$$

$$k = 1$$

\therefore y-axis divides in 1 : 1

OR

- 

$$\begin{aligned} AB &= \sqrt{(4 - 1)^2 + (2 - 7)^2} \\ &= \sqrt{9 + 25} \\ &= \sqrt{34} \end{aligned}$$